PHYS 4A – Spring 2024 – Midterm 1

Question:	1	2	3	4	5	Total
Points:	6	8	8	8	0	30
Bonus Points:	0	0	0	0	5	5
Score:						

1. (6 points) For projectile motion starting and ending at the same height, the distance is given by the formula:

$$d = \frac{v_i^2 \sin 2\theta_i}{g}$$

where v_i is the initial *speed* and θ_i is the angle to the horizontal. A soccer player passes the ball to his teammate 20.0 meters away in a projectile motion. Given that he normally kicks the ball at 20.0 m/s, how long is the ball in the air?

Solution: From the above equation we can calculate the angle θ_i whereby

$$\sin 2\theta_i = 20\,\mathrm{m} \times 9.81\,\mathrm{m/s^2} / \left(20\,\mathrm{m/s}\right)^2 \Rightarrow \theta_i = \frac{1}{2} \arcsin 0.491 = 14.7^\circ$$

The horizontal velocity is $v_i \cos\theta_i = 19.3\,\mathrm{m/s}$ such that $t_{\mathrm{air}} = d/v_x = 1.03\,\mathrm{s}$

2. (8 points) A woman is driving her car and her phone is lying flat on the dashboard. There is a curve coming up and the radius of the curve is 100 meters. Assuming the static friction coefficient between the phone and the dashboard is 0.800, what is the maximum speed she can have when entering into the curve so that the phone doesn't slide. Assume the car's speed before entering the curve and in the curve is constant.

Solution: The maximum friction force that can be achieved by the phone is $\mu_s F_N$. On the other hand, for the phone to be able to make the curve, it will have a centripetal acceleration of $a_c = v^2/R$. The freebody equations of the phone therefore is:

$$\begin{pmatrix} ma_c \\ 0 \end{pmatrix} = \begin{pmatrix} \mu_s F_N \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -mg \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix}$$

We see that in the y-component of the equation $F_N = mg$ and therefore the x-component of the equation gives:

$$mv^2/R = \mu_s mg$$

such that

$$v = \sqrt{100 \times 9.81}\,\mathrm{m/s} = 28.0\,\mathrm{m/s}$$

3. (8 points) In transatlantic air travel between New York and London the flight time on an Airbus 380 from New York to London (eastwards) is 5h30 but the return journey (westwards) takes 6h30. The jetstream, which can be assumed to be a *constant wind* at the airplane's cruising altitude, creates this effect as the airplane flies in the jetstream. The airplane's normal cruising speed with respect to air is 920 km/h. Assuming that the airplane's trajectory and jetstream are completely aligned, calculate the *speed* and *direction* of the jetstream (east or west) as well as the *distance* between New York and London.

Solution: To solve this equation we can be pragmatic and say that the ground speed of the aircraft with respect to the ground is $v_{\text{east}} = 920\,\mathrm{km/h} + v_j$ eastwards and $v_{\text{west}} = 920\,\mathrm{km/h} - v_j$ westwards. The sign of v_j will tell as which direction as we have now chosen *east* to be the positive direction.

Using the times we can write the distance in both directions:

$$d = v_{\text{east}} \times t_{\text{east}} \;\text{ and }\; d = v_{\text{west}} \times t_{\text{west}}$$

using the values given:

$$d = (920\,\text{km/h} + v_j) \times 5.5\text{h} \;\text{ and }\; d = (920\,\text{km/h} - v_j) \times 6.5\text{h}$$

Dividing the two equations to each other we get:

$$1 = \frac{(920\,\text{km/h} + v_j) \times 5.5\text{h}}{(920\,\text{km/h} - v_j) \times 6.5\text{h}}$$

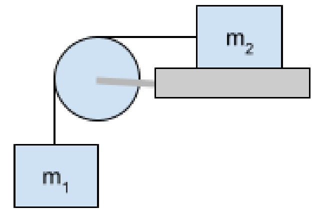
which can be solved as:

$$v_j = 76.7\,\text{km/h} \quad \text{in the eastward direction}$$

and substituting in either one of the equation for d we get:

$$d = 5480\,\text{km}$$

4. The friction coefficients between the table and m_2 are $\mu_s = 0.800$ and $\mu_k = 0.600$. The masses are $m_1 = 10.0\,\text{kg}$ and $m_2 = 10.0\,\text{kg}$. The pulley and the rope are massless.



(a) (4 points) What is the acceleration of m_1?

Solution: We need to ascertain that the system is not static. Assuming $a = 0$, $F_{s_{\max}} = \mu_s m_2 g = 78.5\,\text{N}$ as the normal force on m_2 is just $m_2 g$ and the tension in the rope is equal to $F_{\text{rope}} = m_1 g = 98.1\,\text{N}$. Therefore a static equilibrium is not possible.

We need to use μ_k. We can see that the friction force is $F_k = \mu_k m_2 g = 58.9\,\text{N}$. Then the net force on the system is: $F_{\text{net}} = -m_1 g + F_k = -39.2\,\text{N}$. Therefore:

$$(m_1 + m_2)a = -39.2\,\text{N} \quad \Rightarrow \quad a = -1.96\,\text{m/s}^2$$

(b) (4 points) How would we have to change m_2 for the system to be *stationary*? Calculate the minimum value of m_2 that would achieve this.

Solution: For acceleration to be zero, we need to have $F_s = \mu_s m_2 g$ to be equal to the tension in the rope when m_1 is stationary. If m_1 is stationary, the tension in the rope is $F_r = m_1 g$. Therefore:

$$F_r = F_s \Rightarrow m_1 g = \mu_s m_2 g$$

which means $m_2 = m_1/\mu_s = 12.5\,\text{kg}$

5. (5 points (bonus)) A typical motorcycle has its engine connected *only* to the rear wheel. Assuming that 50% of the combined weight of the motorcycle and the rider is carried by the rear wheel, what is the maximum acceleration the motorcycle can attain if $\mu_s = 0.9$ and $\mu_k = 0.7$.

Solution: If only half of the weight is carried by the rear wheel, then the normal force on the rear wheel is $F_{Nr} = mg/2$ As a rolling wheel has a non-slipping contact surface, we need to use the static friction coefficient. With this normal force, the maximum horizontal force that can be create is $F_h = \mu_s F_{Nr} = 0.45mg$. Therefore the horizontal component of the free body diagram will give $ma_h = F_h = 0.45mg$. The maximum possible acceleration is 0.45g.