## SECTION 5.1 PROBLEM SET: EXPONENTIAL GROWTH AND DECAY FUNCTIONS

Identify each as an exponential, linear, or power function

|  |  |
| --- | --- |
| 1) y = 640 (1.25x) | 2) y = 640 (x1.25) |
| 3) y = 640 (1.25x) | 4) y = 1.05x−2.5 |
| 5) y = 90− (4/5)x | 6) y = 42(0.92x) |
| 7) y = 37(x 0.25) | 8) y = 4(1/3)x |

Indicate if the function represents exponential growth or exponential decay.

|  |  |
| --- | --- |
| 9) y = 127e−0.35t | 10) y = 70 (0.8t) |
| 11) y = 453(1.2t) | 12) y = 16e0.2t |

In each of the following, y is an exponential function of t stated in the form y = aekt where t represents

 time measured in years.. For each:
a. re-express each function in the form y = abt (state the value of b accurate to 4 deicmal places)b. state the annual growth rate or annual decay rate as a percent, accurate to 2 decimal places

|  |  |
| --- | --- |
| 13) y = 127e−0.35t | 14) y = 16e0.4t |
| 15) y = 17250 e0.24t  | 16) y = 4700 e−0.07t |

***SECTION 5.1 PROBLEM SET: EXPONENTIAL GROWTH AND DECAY FUNCTIONS***

Identify if the function represents exponential growth, exponential decay, linear growth, or linear decay.

In each case write the function and find the value at the indicated time.

|  |  |
| --- | --- |
| 17) A house was purchased for $350,000 in the year 2010. The value has been increasing by $7,000 per year. Write the function and find the value of the house after 5 years. | 18) A house was purchased for $350,000 in the year 2010. The value has been increasing at the rate of 2% per year. Write the function and find the value of the house after 5 years. |
| 19) A lab purchases new equipment for $50,000. Its value depreciates over time. The value decreases at the rate of 6% annually. Write the function and find the value after 10 years. | 20) A lab purchases new equipment for $50,000. Its value depreciates over time. The value decreases by $3000 annually. Write the function and find the value after 10 years |
| 21) A population of bats in a cave has 200 bats. The population is increasing by 10 bats annually. Write the function. How many bats live in the cave after 7 years?  | 22) A population of bats in a cave has 200 bats. The population is increasing at the rate of 5% annually. Write the function. How many bats live in the cave after 7 years?  |
| 23) A population of a certain species of bird in a state park has 300 birds. The population is decreasing at the rate of 7% year. Write the function. How many birds are in the population after 6 years? | 24) A population of a certain species of bird in a state park has 300 birds. The population decreases by 20 birds per year. Write the function. How many birds are in the population after 6 years? |

***SECTION 5.1 PROBLEM SET: EXPONENTIAL GROWTH AND DECAY FUNCTIONS***

In problems 25-28, the problem represents exponential growth or decay and states the CONTINUOUS growth rate or continuous decay rate. Write the exponential growth or decay function and find the value at the indicated time.

*Hint: Use the form of the exponential function that is appropriate when the CONTINUOUS growth or decay rate is given.*

|  |  |
| --- | --- |
| 25) A population of 400 microbes increases at the **continuous** growth rate of 26% per day. Write the function and find the number of microbes in the population at the end of 7 days. | 26) The price of a machine needed by a production factory is $28,000. The business expects to replace the machine in 4 years. Due to inflation the price of the machine is increasing at the **continuous** rate of 3.5% per year. Write the function and find the value of the machine 4 years from now. |
| 27) A population of an endangered species consists of 4000 animals of that species. The population is decreasing at the continuous rate of 12% per year. Write the function and find the size of the population at the end of 10 years. | 28) A business buys a computer system for $12000. The value of the system is depreciating and decreases at the continuous rate of 20% per year. Write the function and find the value at the end of 3 years.  |

## SECTION 5.2 PROBLEM SET: GRAPHS AND PROPERTIES OF EXPONENTIAL GROWTH AND DECAY FUNCTIONS

In questions 1-4, let t = time in years and y = the value at time t or y = the size of the population at time t. The domain is the set of non-negative values for t; t ≥ 0, because y represents a physical quantity and negative values for time may not make sense. For each question:

a. Write the formula for the function in the form y = abt

b. Sketch the graph of the function and mark the coordinates of the y-intercept.

|  |  |
| --- | --- |
| 1) A house was purchased for $350,000 in the year 2010. The value has been increasing at the rate of 2% per year.  | 2) A population of a certain species of bird in a state park has 300 birds. The population is decreasing at the rate of 7% year.  |
| 3) A lab buys equipment $50,000. Its value depreciates over time. The value decreases at the rate of 6% annually.  | 4) A population of bats in a cave has 200 bats. The population is increasing at the rate of 5% annually.  |

***SECTION 5.2 PROBLEM SET: GRAPHS AND PROPERTIES OF EXPONENTIAL
GROWTH AND DECAY FUNCTIONS***

In questions 5-8, let t = time in years and y = the value at time t or y = the size of the population at time t. The domain is the set of non-negative values for t; t ≥ 0, because y represents a physical quantity and negative values for time may not make sense. For each question:

a. Write the formula for the function in the form y = aekt

b. Sketch the graph of the function and mark the coordinates of the y-intercept.

|  |  |
| --- | --- |
| 5) A population of 400 microbes increases at the **continuous** growth rate of 26% per day.   | 6) The price of a machine needed by a production factory is $28,000. Due to inflation the price of the machine is increasing at the **continuous** rate of 3.5% per year.  |
|  7) A population of an endangered species consists of 4000 animals of that species. The population is decreasing at the **continuous** rate of 12% per year.  |  8) A business buys a computer system for $12000. The value of the system is depreciating and decreases at the **continuous** rate of 20% per year.  |

***SECTION 5.2 PROBLEM SET: GRAPHS AND PROPERTIES OF EXPONENTIAL
GROWTH AND DECAY FUNCTIONS***

For questions 9-12

a. sketch a graph of exponential function

b. list the coordinates of the y intercept

c. state the equation of any asymptotes and state the whether the function approaches the asymptote as
x →∞ or as x→ −∞

d. State the domain and range

|  |  |
| --- | --- |
| 9) y = 10(1.5x)  | 6) y = 10(e1.2x)   |
| 11) y = 32(0.75x)  | 12) y = 200(e−.5x) |

## SECTION 5.3 PROBLEM SET: LOGARITHMS AND LOGARITHMIC FUNCTIONS

 Rewrite each of these exponential expressions in logarithmic form:

|  |  |
| --- | --- |
| 1) 34=81 | 2) 105=100,000 |
| 3) 5−2=0.04 | 4) 4−1=0.25 |
| 5) 161/4=2 | 6) 91/2=3 |

 Rewrite each of these logarithmic expressions in exponential form:

|  |  |
| --- | --- |
| 7) log 5 625 = 4 | 8) log 2 (1/32) = −5 |
| 9) log 11 1331 = 3 | 10) log 10 0.0001 = −4 |
| 11) log 64 4 = 1/3 | 12)   |

If the expression is in exponential form, rewrite it in logarithmic form.
If the expression is in logarithmic form, rewrite it in exponential form.

|  |  |
| --- | --- |
| 13) 5x=15625 | 14) x = 93 |
| 15) log 5 125 = x | 16) log 3 x = 5 |
| 17) log 10 y = 4 | 18) ex = 10 |
| 19) ln x = −1  | 20) e5 = y |

***SECTION 5.3 PROBLEM SET: LOGARITHMS AND LOGARITHMIC FUNCTIONS***

For each equation, rewrite in exponential form and solve for x.

|  |  |
| --- | --- |
| 21) log5 (x) = 3  | 22) log2 (x) = −2  |
| 23) log10 (x) = –3  | 24) log3 (x) = 6 |
| 25) log25 (x) = 1/2 | 26) log64 (x) = 1/3  |

Evaluate without using your calculator.

|  |  |
| --- | --- |
| 27) ln   | 28) ln   |
| 29) ln e10  | 30)log10 (10*e* ) |

 For problems 31 – 38: Evaluate using your calculator. Use the change of base formula if needed

|  |  |
| --- | --- |
| 31) log 20  | 32) ln 42  |
| 33) ln 2.9 | 34) log 0.5  |
| 35) log4 36 | 36) log 7 100 |
| 37) log1.05 3.5 | 38) log 1.067 2 |

## SECTION 5.4 PROBLEM SET: GRAPHS AND PROPERIES OF LOGARITHMIC FUNCTIONS

Questions 1 – 3: For each of the following functions

a. Sketch a reasonably accurate graph showing the shape of the graph of the function

b. State the domain

c. State the range

d. State whether the graph has a vertical asymptote or a horizontal asymptote and write the equation of that asymptote

e. Does the graph have an x-intercept or a y-intercept asymptote? Write the coordinates of the
x-intercept or the y-intercept.

|  |  |
| --- | --- |
| 1) y = ln xa. Sketch the graph below | b. domain:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_c. range:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_d. Is the asymptote horizontal or vertical? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Equation of the asymptote:\_\_\_\_\_\_\_\_\_\_ e. Coordinates of x intercept or y intercept:\_\_\_\_\_\_\_ |
| 2) y = log xa. Sketch the graph below | b. domain:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_c. range:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_d. Is the asymptote horizontal or vertical?  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Equation of the asymptote:\_\_\_\_\_\_\_\_\_\_ e. Coordinates of x intercept or y intercept:\_\_\_\_\_\_ |
| 3) y = log0.8xa. Sketch the graph below | b. domain:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_c. range:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_d. Is the asymptote horizontal or vertical? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_Equation of the asymptote:\_\_\_\_\_\_\_\_\_\_ e. Coordinates of x intercept or y intercept:\_\_\_\_\_\_ |

***SECTION 5.4 PROBLEM SET: GRAPHS AND PROPERIES OF LOGARITHMIC FUNCTIONS***

Questions 4 - 5: For the pair of inverse functions y = ex and y = ln x

a. Sketch a reasonably accurate graph showing the shape of the graph of the function

b. State the domain

c. State the range

d. State whether the graph has a vertical asymptote or a horizontal asymptote and write the equation of that asymptote

e. Does the graph have an x-intercept or a y-intercept asymptote? Write the coordinates of the x-intercept or the y-intercept.

|  |  |
| --- | --- |
| 4) y = *e* xSketch the graph below | 5) y = ln xSketch the graph below |
| b. domain:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_c. range:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_d. Is the asymptote horizontal or vertical? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Equation of the asymptote:\_\_\_\_\_\_\_\_\_\_ e. Coordinates of x intercept or y intercept: \_\_\_\_\_\_\_ | b. domain:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_c. range:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_d. Is the asymptote horizontal or vertical? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Equation of the asymptote:\_\_\_\_\_\_\_\_\_\_\_\_\_ e. Coordinates of x intercept or y intercept: \_\_\_\_\_\_\_ |

***SECTION 5.4 PROBLEM SET: GRAPHS AND PROPERIES OF LOGARITHMIC FUNCTIONS***

**Questions 6-11: Match the graph with the function.**
Choose the function from the list below and write it on the line underneath the graph.

*Hint: To match the function and the graph, identify these properties of the graph and function*

* *Is the function increasing decreasing?*
* *Examine the asymptote*
* *Determine the x or y intercept*

y = 3(2x) y = 5(0.4x) y = log2(x) y = log 1/2 (x) y = 3e−0.6x y = 5e0.3x



## SECTION 5.5 PROBLEM SET: APPLICATIONS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

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| --- | --- |
|  1) An investment’s value is rising at the rate of 5% per year. The initial value of the investment is $20,000 in 2016. a. Write the function that gives the value of the investment as a function of time t in years after 2016. b. Find the value of the investment in 2028c. When will the value be $30,000? | 2) The population of a city is increasing at the rate of 2.3% per year, since the year 2000. Its population in 2010 was 137,000 people. Find the population of the city in the year 2000. |
| 3) The value of a piece of industrial equipment depreciates after it is purchased. Suppose that the depreciation follows an exponential decay model. The value of the equipment at the end of 8 years is $30,000 and its value has been decreasing at the rate of 7.5% per year. Find the initial value of the equipment when it was purchased. | 4) An investment has been losing money. Its value has been decreasing at the rate of 3.2% per year. The initial value of the investment was $75,000 in 2010. a. Write the function that gives the value of the investment as a function of time t in years after 2010. b. If the investment’s value continues to decrease at this rate, find the value of the investment in 2020.  |

***SECTION 5.5 PROBLEM SET: APPLICATIONS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS***

|  |  |
| --- | --- |
| 5) A social media site has 275 members initially. The number of members has been increasing exponentially according to the function  y = 275e0.21t, where t is the number of months since the site’s initial launch. How many months does it take until the site has 5000 members? *State answer to the nearest tenth of a month (1 decimal place).* | 6) A city has a population of 62000 people in the year 2000. Due to high unemployment, the city’s population has been decreasing at the rate of 2% per year. Using this model, find the population of this city in 2016. |
| 7) A city has a population of 87,000 people in the year 2000. The city’s population has been increasing at the rate of 1.5% per year. How many years does it take until the population reaches 100,000 people? | 8) An investment of $50,000 is in increasing in value at the rate of 6.3% per year. How many years does it take until the investment is worth $70,000? |

***SECTION 5.5 PROBLEM SET: APPLICATIONS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS***

|  |  |
| --- | --- |
| 9) A city has a population of 50,000 people in the year 2000. The city’s population increases at a constant percentage rate. Fifteen years later, in 2015, the population of this city was 70,000. Find the annual percentage growth rate. | 10) 200 mg of a medication is administered to a patient. After 3 hours, only 100 mg remains in the bloodstream. Using an exponential decay model, find the hourly decay rate. |
| 11) An investment is losing money at a constant percentage rate per year. The investment was initially worth $25,000 but is worth only $20,000 after 4 years. Find the percentage rate at which the investment is losing value each year (that is, find the annual decay rate). | 12) Using the information in question 11, how many years does it take until the investment is worth only half of its initial value? |

***SECTION 5.5 PROBLEM SET: APPLICATIONS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS***

 For question 13:

* if the function is given in the form y = a*e*kt, rewrite it in the form y = abt.
* if the function is given in the form y = abt, rewrite it in the form y = a*e*kt.

|  |  |
| --- | --- |
| 13a) y = 7900*e*0. 472t. Write in the form y = abt | 13b) y = 4567(0.67t) Write in the form y = a*e*kt |
| 13c) y = 18720(1.47t) Write in the form y = a*e*kt | 13d) y = 1200*e*−0. 078t. Write in the form y = abt |

## SECTION 5.6 PROBLEM SET: CHAPTER REVIEW

1) The value of a new boat depreciates after it is purchased. The value of the boat 7 years after it was purchased is $25,000 and its value has been decreasing at the rate of 8.2% per year.

a. Find the initial value of the boat when it was purchased.

b. How many years after it was purchased will the boat’s value be $20,000?

c. What was its value 3 years after the boat was purchased?

2) Tony invested $40,000 in 2010; unfortunately his investment has been losing value at the rate of 2.7% per year.

a. Write the function that gives the value of the investment as a function of time t in years after 2010.

b. Find the value of the investment in 2020, if its value continues to decrease at this rate.

c. In what year will the investment be worth half its original value?

3) Rosa invested $25,000 in 2005; its value has been increasing at the rate of 6.4% annually.

a. Write the function that gives the value of the investment as a function of time t in years after 2005.

b. Find the value of the investment in 2025.

4) The population of a city is increasing at the rate of 3.2% per year, since the year 2000. Its population in 2015 was 235,000 people.

a. Find the population of the city in the year 2000.

b. In what year with the population be 250, 000 if it continues to grow at this rate.

c. What was the population of this city in the year 2008?

5) The population of an endangered species has only 5000 animals now. Its population has been decreasing at the rate of 12% per year.

a. If the population continues to decrease at this rate, how many animals will be in this population 4 years from now.

b. In what year will there be only 2000 animals remaining in this population?

6) 300 mg of a medication is administered to a patient. After 5 hours, only 80 mg remains in the bloodstream.

a. Using an exponential decay model, find the hourly decay rate.

b. How many hours after the 300 mg dose of medication was administered was there 125 mg in the bloodstream

c. How much medication remains in the bloodstream after 8 hours?

7) If y = 240bt and y = 600 when t = 6 years, find the annual growth rate. State your answer as a percent.

8) If the function is given in the form y = a*e*kt, rewrite it in the form y = abt.
 If the function is given in the form y = abt, rewrite it in the form y = a*e*kt.

 a. y = 375000(1.125t) b. y = 5400*e*0. 127t c. y = 230*e*−0**.** 62t d. y = 3600(0.42t)