① $y = mx + b$ (or $y = b + mx$) slope intercept form

ⓐ $b = 20$ $m = \$.05/\text{minute}$

$y = .05x + 20$ or equivalently $y = 20 + .05x$

ⓑ $b = 243$ $y = \$.19/\text{mile}$

$y = .19x + 243$ or equivalently $y = 243 + .19x$

ⓒ $b = 66000$ $y = \$-3000/\text{year}$

$y = -3000x + 66000$ or equivalently $y = 66000 - 3000x$

② $y - y_1 = m(x - x_1)$ point slope form

ⓐ $(x_1, y_1) = (3, 10)$ (3 books, \$10)

$m = \frac{\Delta y}{\Delta x} = \frac{\text{change in cost}}{\text{change in \# of books}} = \frac{+\$2}{+1 \text{ book}}$ so $m = 2$

$y - 10 = 2(x - 3)$ simplifies to $y = 4 + 2x$

ⓑ $(x_1, y_1) = (120, 200)$ (120 cupcakes, \$200)

$m = \frac{\Delta y}{\Delta x} = \frac{\text{change in cost}}{\text{change in \# of cupcakes}} = \frac{+\$9}{+12 \text{ cupcakes}} = \frac{9}{12}$ so $m = .75$

$y - 200 = .75(x - 120)$ simplifies to $y = 110 + .75x$

ⓒ $(x_1, y_1) = (15, 200)$ (\$15, 200 frames)

$m = \frac{\Delta y}{\Delta x} = \frac{\text{change in \# of frames}}{\text{change in price}} = \frac{-60 \text{ frames}}{+\$5} = -\frac{60}{5}$ so $m = -12$

$y - 200 = -12(x - 15)$ simplifies to $y = 380 - 12x$

ⓓ $(x_1, y_1) = (50, 30)$ (50°F, 30 gallons)

$m = \frac{\Delta y}{\Delta x} = \frac{\text{change in quantity of ice cream}}{\text{change in temperature}} = \frac{+1 \text{ gallon}}{+2°F}$ so $m = .5$

$y - 30 = .5(x - 50)$ simplifies to $y = 5 + .5x$

② continued $y - y_1 = m(x - x_1)$ point slope form

ⓔ $(x_1, y_1) = (24, 1000)$ (\$24, 1000 clocks)

$$m = \frac{\Delta y}{\Delta x} = \frac{\text{change in \# of clocks}}{\text{change in price}} = \frac{+45 \text{ clocks}}{-\$1 \text{ price}} \quad \text{so } m = -45$$

$y - 1000 = -45(x - 24)$ simplifies to $y = 2080 - 45x$

③ $m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$ to find slope

$y - y_1 = m(x - x_1)$ point slope form

ⓐ $(x_1, y_1) = (10, 350)$
$(x_2, y_2) = (40, 950)$

or

x bracelets	\$y cost
10	350
40	950

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(950 - 350)}{(40 - 10)} = \frac{600}{30} = 20$$

$y - 350 = 20(x - 10)$ simplifies to $y = 150 + 20x$

ⓑ $(x_1, y_1) = (1000, 20000)$
$(x_2, y_2) = (3000, 50000)$

or

x tee shirts	\$y cost
1000	20000
3000	50000

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(50000 - 20000)}{(3000 - 1000)} = \frac{30000}{2000} = 15$$

$y - 20000 = 15(x - 1000)$ simplifies to $y = 5000 + 15x$

ⓒ $(x_1, y_1) = (5, 50000)$
$(x_2, y_2) = (8, 35000)$

or

x years	\$y value
5	50000
8	35000

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{35000 - 50000}{8 - 5}$$

so $m = -\dfrac{15000}{3} = -5000$ ← slope tells us the value is decreasing at the rate of \$5000/year.

$y - 50000 = -5000(x - 5)$

simplifies to $y = 75000 - 5000x$

Cost, Revenue, Profit

4 a $y = C(x) = 300 + 5x$ cost function

b $C(60) = 300 + 5(60) = \$600$ cost to make 60 pizzas

Tony needs to charge $\dfrac{\$600}{60 \text{ pizzas}} = \$10$ per pizza

in order to break even, or over \$10 per pizza to make a profit

c $C(x) = 300 + 5x$ cost function

$R(x) = 15x$ revenue function as revenue = (price)(quantity)

Profit: $P(x) = R(x) - C(x) = 15x - (300 + 5x) = 10x - 300$

To break even Revenue = Cost

$$R(x) = C(x)$$
$$15x = 300 + 5x$$
$$10x = 300$$
$$x = 30 \text{ pizzas}$$

d If price = \$15 and $x = 20$ pizzas

Revenue = $R(20) = (15)(20) = 300$

Cost = $C(20) = 300 + 5(20) = 400$

$P(20) = R(20) - C(20) = 300 - 400 = -\$100$ \$100 LOSS

e If price = \$15 and $x = 50$ pizzas

Revenue = $R(50) = 15(50) = 750$

Cost = $C(50) = 300 + 5(50) = 550$

$P(50) = R(50) - C(50) = 750 - 550 = \$200$ \$200 PROFIT

Cost, Revenue, Profit

⑤ ⓐ find cost function

$(x_1, y_1) = (100, 2000)$

$(x_2, y_2) = (500, 5000)$

OR

X phone cases	\$Y cost
100	2000
500	5000

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(5000-2000)}{(500-100)} = \frac{3000}{400} = 7.50$$

$$y - 2000 = 7.50(x - 100)$$

Cost function simplified is: $y = C(x) = 1250 + 7.50x$

ⓑ revenue function = (price)(quantity)

$y = R(x) = 10x$

profit = revenue − cost

$P(x) = R(x) - C(x) = 10x - (1250 + 7.50x)$

$P(x) = 2.50x - 1250$ when simplified

ⓓ Break-Even Profit = 0 ; Revenue = Cost

$R(x) = C(x)$

$10x = 1250 + 7.50x$

$2.50x = 1250$

$x = 500$ cell phone cases

$C(500) = 1250 + 7.50(500)$

$C(500) = \$5000$

$R(500) = 10(500)$

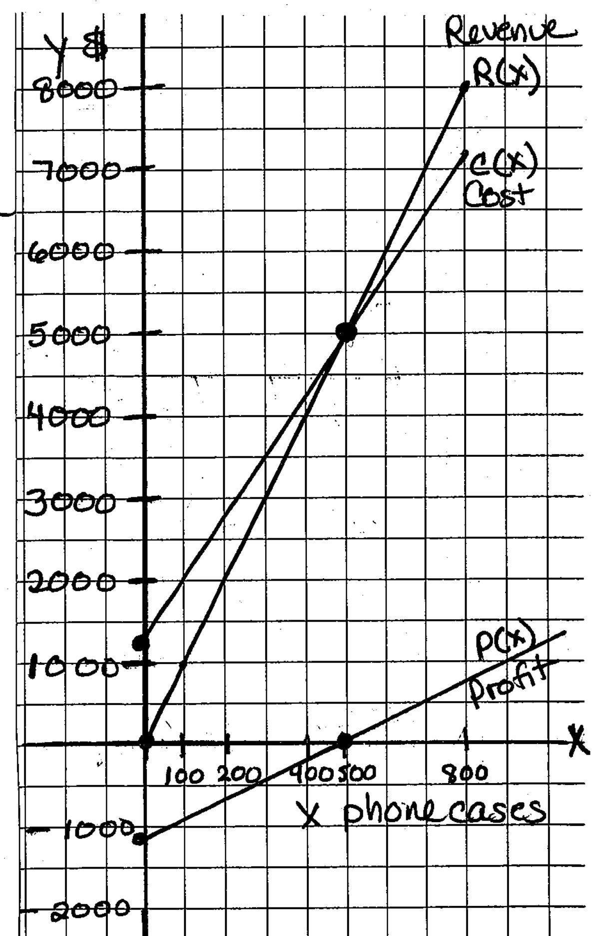
$R(500) = \$5000$

ⓒ Graph ⟶

$R(x)$ passes through $(0,0)$ $(500, 5000)$

$C(x)$ passes through $(0, 1250)$ $(500, 5000)$

$P(x)$ passes through $(0, -1250)$ $(500, 0)$



Cost, Revenue, Profit

6. cost function

$(x_1, y_1) = (10, 350)$ OR

$(x_2, y_2) = (40, 950)$

x bracelets	y \$ cost
10	350
40	950

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{950-350}{40-10} = \frac{600}{30} = 20$$

$y - 350 = 20(x-10)$ so $y = C(x) = 150 + 20x$ cost function

revenue function = (price)(quantity)

$y = R(x) = 27.50x$ revenue function

Break Even when revenue = cost, then profit = 0

$R(x) = C(x)$

$27.50x = 150 + 20x$

$7.5x = 150$

$x = 20$ bracelets

When 20 bracelets are produced and sold,

$R(20) = 27.50(20) =$ \$550 revenue

$C(20) = 150 + 20(20) =$ \$550 cost

$P(20) = R(20) - C(20) =$ \$0 profit

Cost, Revenue, Profit

7 (a) Cost Function: $y = C(x) = 4000 + 7x$

Revenue Function: $y = R(x) = 12x$

Profit Function: $y = P(x) = R(x) - C(x)$

$$y = P(x) = 12x - (4000 + 7x)$$

$$y = P(x) = 5x - 4000$$

(b) When $x = 600$ shower heads:

cost $C(600) = 4000 + 7(600) = \$8200$

revenue $R(600) = 12(600) = \$7200$

$P(600) = R(600) - C(600) = 7200 - 8200 = -\$1000$

\$1000 loss

(c) When $x = 1200$ shower heads

cost $C(1200) = 4000 + 7(1200) = \$12400$

revenue $R(1200) = 12(1200) = \$14400$

$P(1200) = R(1200) - C(1200) = 14400 - 12400 = \$2000$

\$2000 profit

(d) Breakeven: Revenue = Cost when profit = 0

$$R(x) = C(x)$$

$$12x = 4000 + 7x$$

$$5x = 4000$$

$x = 800$ shower heads

At $x = 800$: $C(800) = 4000 + 7(8000) = \$9600$

$R(800) = 12(8000) = \$9600$

so profit $= \$0$

8) Supply and Demand

(a) Supply Function

$(x_1, y_1) = (15, 125)$

$(x_2, y_2) = (22, 216)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{216-125}{22-15} = \frac{91}{7} = 13$

$y - 125 = 13(x - 15)$

simplifies to

$y = S(x) = -70 + 13x$

Demand Function

$(x_1, y_1) = (15, 200)$

$m = \frac{\Delta y}{\Delta x} = \frac{\Delta \text{frames}}{\Delta \text{price}} = \frac{-60}{+5} = -12$

$y - 200 = -12(x - 15)$

simplifies to

$y = D(x) = 380 - 12x$

(b) At equilibrium: Supply = Demand

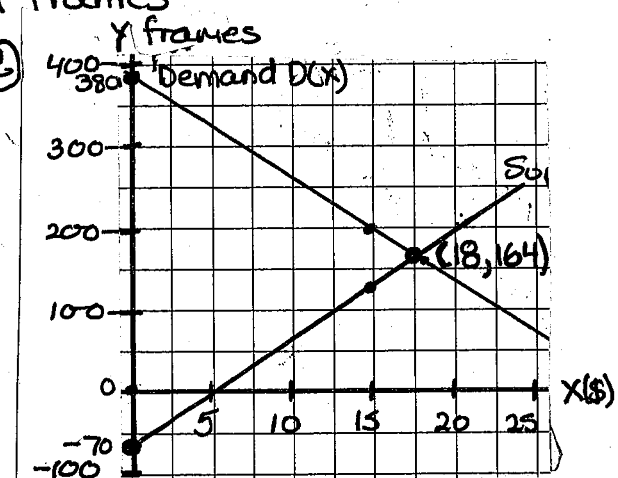
$$S(x) = D(x)$$
$$-70 + 13x = 380 - 12x$$
$$25x = 450$$
$$x = \$18 \text{ equilibrium price}$$

When price = $18

$S(18) = -70 + 13(18) = 164$ frames

$D(18) = 380 - 12(18) = 164$ frames

(c)



Demand D(x) passes through (0,380), (15,200) (18,164)

Supply S(x) passes through (0,-70), (15,125) (18,164)

Plot points to graph lines

(d) If price x = $16

$S(16) = -70 + 13(16) = 138$ frames

$D(16) = 380 - 12(16) = 188$ frames

Demand exceeds supply

Supply is less than demand

50 more frames could be sold than are produced

(e) If x = $21 price

$S(21) = -70 + 13(21) = 203$ frames

$D(21) = 380 - 12(21) = 128$ frames

Supply exceeds demand

$203 - 128 = 75$ more frames are made than can be sold

#9 Supply and Demand

Wooden Puzzles

Supply: $y = S(x) = 80x - 900$

Demand: $y = D(x) = 1200 - 70x$

x = price
y = number of puzzles

ⓐ $S(15) = 80(15) - 900 = 1200 - 900 = 300$ puzzles produced
$D(15) = 1200 - 70(15) = 1200 - 1050 = 150$ puzzles purchased
Supply exceeds demand

ⓑ $S(12) = 80(12) - 900 = 960 - 900 = 60$ puzzles produced
$D(12) = 1200 - 70(12) = 1200 - 840 = 360$ puzzles purchased
Demand exceeds supply

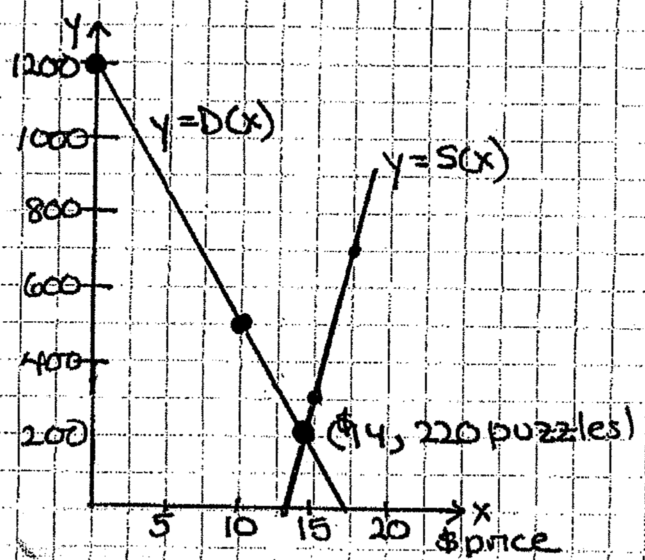
ⓒ Equilibrium

$$S(x) = D(x)$$
$$80x - 900 = 1200 - 70x$$
$$150x = 2100$$
$$x = \frac{2100}{150} = \$14$$

$D(14) = 1200 - 70(14) = 220$ puzzles
$S(14) = 80(14) - 900 = 220$ puzzles

When the prices is \$14, supply and demand are in equilibrium. 220 puzzles are produced by suppliers and bought by consumers.

ⓓ Graph



To graph the lines pick 2 points on each line

D(x): $x = 0$ $D(0) = 1200$
$x = 10$ $D(10) = 500$

S(x): $x = 15$ $S(15) = 300$
$x = 20$ $S(20) = 700$

ⓔ For every \$1 increase in price, suppliers are willing to produce 80 additional puzzles

ⓕ For every \$1 increase in price, consumers are willing to purchase 70 fewer puzzles.

(XX10) Supply and Demand

Cardboard Puzzles — Supply $y = S(x) = 60x - 120$ — Demand $y = D(x) = 480 - 40x$

ⓐ $S(4) = 60(4) - 120 = 120$ puzzles produced by suppliers
$D(4) = 480 - 40(4) = 320$ puzzles demanded by consumers
Demand exceeds supply

ⓑ $S(7) = 60(7) - 120 = 300$ puzzles produced
$D(7) = 480 - 40(7) = 200$ puzzles purchased
Supply exceeds demand

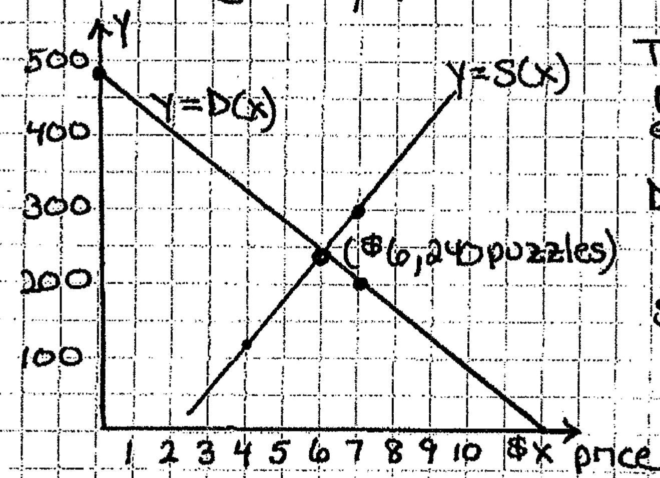
ⓒ Equilibrium

$$S(x) = D(x)$$
$$60x - 120 = 480 - 40x$$
$$100x = 600$$
$$x = \frac{600}{100} = \$6$$

$D(6) = 480 - 40(6) = 240$ puzzles
$S(6) = 60(6) - 120 = 240$ puzzles

When the price is \$6, supply and demand are in equilibrium. 240 puzzles are produced by suppliers and bought by producers

ⓓ Graph



To graph the lines pick 2 points on each line

D(x): $x = 0$ $D(0) = 480$
$x = 7$ $D(7) = 200$

S(x): $x = 7$ $S(7) = 300$
$x = 4$ $S(4) = 120$

ⓔ For every \$1 increase in price, the number of puzzles suppliers produce increases by 60 puzzles.

ⓕ For every \$1 increase in price, the number of puzzles consumers are willing to purchase decreases by 40 puzzles.

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(11) (a) Company A: $y = f(x) = 20 + .05x$

(b) Company B: $y = g(x) = 5 + .10x$

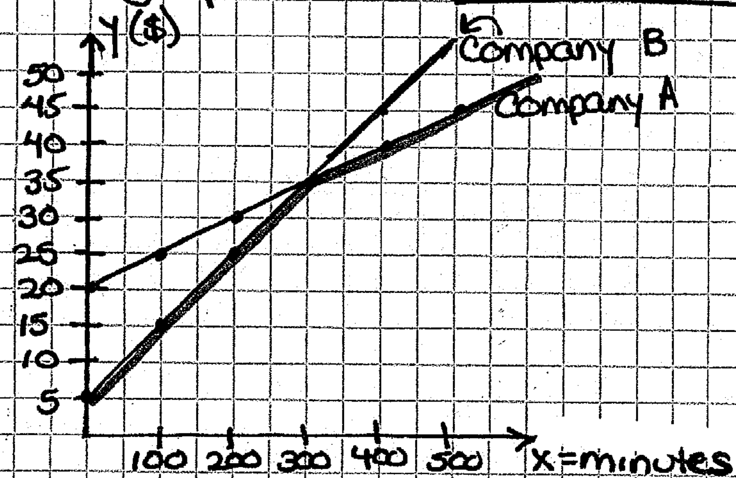
(c) $f(x) = g(x)$

$$20 + .05x = 5 + .10x$$
$$15 = .05x$$
$$300 = x$$

Their costs are the same when $x = 300$ minutes of phone calls; the cost is \$35.

$f(300) = 35$
$g(300) = 35$

(d)(e) Use a graph:



Graph $f(x)$ for A using $(0, 20)$ $(300, 35)$

Graph $g(x)$ for B using $(0, 5)$ $(300, 35)$

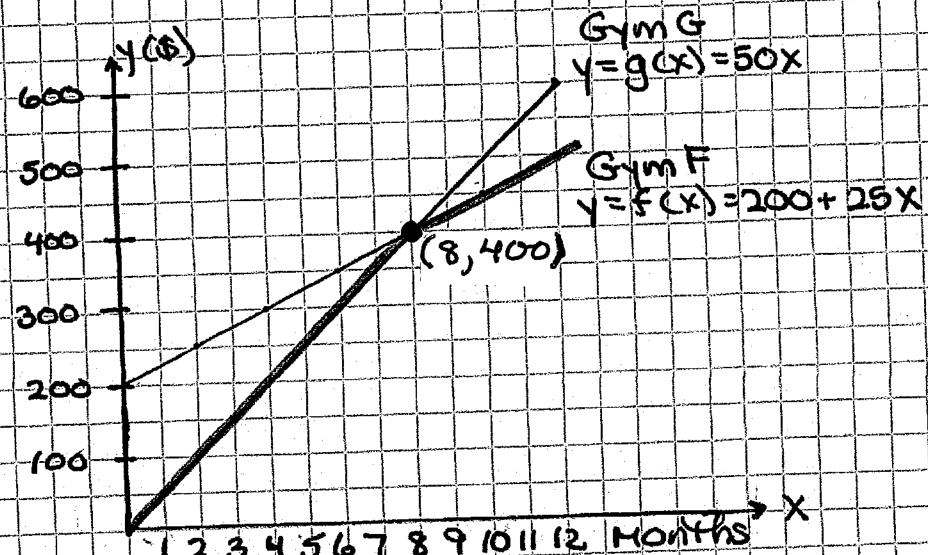
(d) Company A costs less if $x > 300$ minutes

(e) Company B costs less if $x < 300$ minutes

⑫ ⓐ Gym F: $f(x) = 200 + 25x$

Gym G: $g(x) = 50x$

* ⓑ



ⓒ $f(x) = g(x)$

$\cancel{200 + 25x} = 50x$

$200 = 25x$

$8 = x$ Costs are equal at x = 8 months
cost is \$400
(8, 400) is point of intersection

ⓓ Gym G costs less to join if joining for under 8 months. After 8 months, Gym F becomes less expensive to join when looking at total cost over the time period of membership.

* To do the graphs: plot 2 points on each graph
Its easiest to find the intersection point (done in ⓒ) before graphing. Otherwise use other points

Gym G: g(x) passes through (0, 0) (2, 100) (4, 200) (8, 400)

Gym F: f(x) passes through (0, 200) (4, 300) (8, 400)

(13) (a) F (FlashNet) $f(x) = 2500 + .06x$

G (Galaxy) $g(x) = .10x$

H (High Speed) $h(x) = 4000 = 4000 + 0x$

(b) $f(x) = g(x)$

$2500 + .06x = .10x$

$2500 = .04x$

$62500 = x$ sales

$6250 = y$ wages

$(62500, 6250)$

$f(x) = h(x)$

$2500 + .06x = 4000$

$.06x = 1500$

$x = 25000$ sales

$y = 4000$ wages

$(25000, 4000)$

$g(x) = h(x)$

$.10x = 4000$

$x = 40000$ sales

$y = 4000$ wages

$(40000, 4000)$

(c) For sales under \$25000, company H pays the most, \$4000 per month.

For sales between \$25,000 and \$62500 company F pays the most: \$2500 + 6% of sales

For sales above \$62500, company G pays the most: 10% of sales with no base salary