

**CHAPTERS 8 and 9 PROBABILITY**  
**SECTION 8.1 PROBABILITY: EVENTS AND PROBABILITIES**

**NOTE: Some but not all examples in these notes will be done in class as we learn the probability concepts in Chapters 8 and 9.**

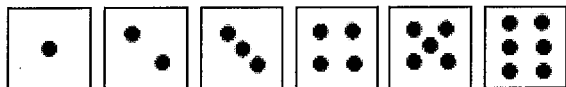
**PROBABILITY:** A probability is a number between 0 and 1, inclusive, that states the long-run relative frequency, likelihood, or chance that an outcome will happen.

**SECTION 8.1: SAMPLE SPACES AND BASIC PROBABILITY**

**EVENT:** An outcome (called a simple event) or a combination of outcomes (called a compound event)

**SAMPLE SPACE:** Set of all possible simple events

**EXAMPLE 1: Rolling 1 die: Sample Space:  $S = \{ \underline{\hspace{2cm}} \}$**



**EXAMPLE 2: A coin is tossed twice.**



Assume the coin is a fair coin - it has equal probability of landing on Head (H) or Tail (T).  
Write the sample space.



Find the probability of getting at least one head.

**EXAMPLE 3: A coin is tossed three times.**



Assume the coin is a fair coin - it has equal probability of landing on Head (H) or Tail (T).  
Write the sample space.



3a. Find the probability that at least one head is obtained

3b. Find the probability that all three tosses have the same outcome

3c. Find the probability that the first and third tosses have the same outcome.



**EXAMPLE 4:** Two dice are tossed.

Find the probability of getting a sum of 8

Write the sample space.

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)  
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)  
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)  
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)  
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)  
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

Find the probability of getting a sum of at least 8

Find the probability of getting a “double”

### Notation

- $P(E) = 0.8$  is read as “the probability of event E is 0.8”
  - P stands for probability
  - ( ) is read “of”; it surrounds the event.
  - The event is written inside the parentheses
  - The value of the probability is on the other side of the equals sign.
- For probability our book uses  $E^c$  to represent the complement of an event.
  - $E^c$  means event E does NOT happen
  - This notation for complement is different notation than our book used for set complements:  $\bar{S}$ .

## SECTION 8.2: ADDITION RULE, COMPLEMENT RULE

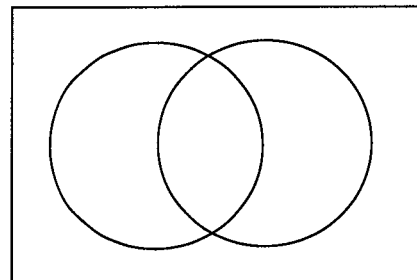
**PROBABILITY RULE FOR COMPLEMENTS:  $P(E^c) = 1 - P(E)$**

**EXAMPLE 5:** Two dice are tossed. Find the probability that the outcome is NOT a double

**ADDITION RULE FOR “OR” (union U) events:**

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$P(E \text{ OR } F) = P(E) + P(F) - P(E \text{ AND } F)$$



**EXAMPLE 6:** Two dice are tossed.

Find the probability of getting a sum of 8 OR a double

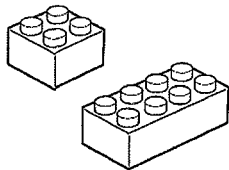
Find the probability of getting a sum of 7 OR a double

**MUTUALLY EXCLUSIVE EVENTS:** Two events are mutually exclusive if they can not both happen. (We called this DISJOINT with sets).  $P(E \cap F) = P(E \text{ AND } F) = 0$

When rolling two dice, the events of “getting a sum of 7”, “getting a double” are mutually exclusive because they can not both happen when rolling two dice

$$P(\text{getting a sum of } 7 \cap \text{getting a double}) = P(\text{getting a sum of } 7 \text{ AND getting a double}) = 0$$

**EXAMPLE 7:** A box of 25 Lego blocks contains:



2 yellow square blocks  
4 blue square blocks  
4 green square blocks

3 yellow rectangular blocks  
8 blue rectangular blocks  
4 green rectangular blocks

Y: yellow
B: blue
G: green
S: square
R: rectangle

A child randomly selects one block at random.

Find  $P(B)$ ,  $P(S)$ ,  $P(B \cap S)$ ,  $P(B \cup S)$ ,  $P((B \cap S)^c)$

**EXAMPLE 8: ADDITION RULE**

In a certain town: 70% of households have Cable TV (*event C*)

55% of households have Netflix (*event N*)

These figures include the fact that 42% of households subscribe to both.

Find the probability that a person subscribes to Cable TV **or** Netflix

**OPTIONAL EXAMPLE 9: SEE SECTION 8.2 IN TEXTBOOK IF NOT DONE IN CLASS**

Mr. Washington is seeking a community college instructor position. His employment depends on two conditions – whether the board approves the position and whether the hiring committee selects him.

There is an 80% chance the board will approve the position.

There is a 70% chance that the hiring committee will select him.

There is a 90% chance that at least one of these will happen.

Find the probability that he will be hired.

A contingency table displays data for two variables. This table shows the number of individuals or items in each category. We can use the data in the table to find probabilities.

**All probabilities EXCEPT conditional probabilities have the grand total in the denominator**

**Conditional Probabilities:** The condition limits you to a particular row or column in the table.

Condition says "IF" we look only at a particular row or column, find the probability

The **denominator will be the total for the row or column** in the table that corresponds to the condition

**EXAMPLE 10:** A large car dealership examined a sample of vehicles sold or leased in the past year. Data is classified by type (**car, SUV, van, truck**) and by whether they were a sale of a **new** or **used** vehicle or whether the vehicle was **leased**.

	Car (C)	SUV (S)	Van (V)	Truck(T)	Total
New vehicle sale (N)	86	25	21	38	170
Used vehicle sale (U)	39	13	4	22	78
Vehicle Lease (L)	34	12	6	0	52
Total	159	50	31	60	300

Suppose a vehicle in the sample is randomly selected to review its sales or lease papers.

- Find the probability that the vehicle was leased.
- Find the probability that a vehicle is a truck.
- Find the probability that a vehicle is NOT a truck.
- Find the probability that the vehicle was a car AND was leased.
- Find the probability that a vehicle was used IF (*given that*) it was a van.
- Find the probability that the vehicle was a van IF (*given that*) it was used.

**Addition Rule for OR Events:**  $P(E \text{ or } F) = P(E) + P(F) - P(E \text{ and } F)$   
 $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

- Find the probability that the vehicle was new OR a van.

- Find the probability that the vehicle was leased OR a truck.

**Mutually Exclusive Events:**  $P(E \text{ and } F) = 0$        $P(E \cap F) = 0$

- Are events T, L mutually exclusive?

- Are events N, V mutually exclusive?

## **SECTION 8.3 : PROBABILITY TREES AND PROBABILITY WITH COMBINATIONS**

TREE DIAGRAMS are a useful tool in organizing and solving probability problems

Each complete path through the tree represents a separate mutually exclusive outcome in the sample space.

1. Draw a tree representing the possible mutually exclusive outcomes
2. Assign conditional probabilities along the branches of the tree
3. Multiply probabilities along each complete path through the tree to find probabilities of each "AND" outcome in the sample space.
4. Add probabilities for the appropriate paths of a tree to find the probability of a compound OR event.

**EXAMPLE 11:** *From Chapter 3 Section 3.7 Tree diagrams in Illowsky, B., & Dean, S. Collaborative Statistics. Connexions, Dec. 5, 2008. <http://cnx.org/content/col10522/1.29>*

**An urn contains 11 marbles, 3 Yellow and 8 Blue. We randomly select 2 marbles from the urn.**

**Select 2 marbles WITH REPLACEMENT:**

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**Select 2 marbles WITHOUT REPLACEMENT**

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**FOR SELECTION WITHOUT REPLACEMENT**

(1) Use the tree to find the probability of getting one marble of each color

(2) Use combinations to find the probability of getting one marble of each color.

### SECTION 8.3 : PROBABILITY WITH COMBINATIONS

**We can use probability with combinations instead of a tree when we are making selections without replacement.**

**EXAMPLE 12:** A college club has 11 members. 7 are part time students and 4 are full time students.

A committee of 5 people is to be formed from the members of this club.

a. Find the probability that a committee of 5 people consists of 3 part time and 2 full time students.

b. Find the probability that a committee of 5 people contains 2 or 3 part time students and the rest full time students.

**EXAMPLE 13:** A librarian has 5 adult fiction books, 6 adult non-fiction books, and 8 children's books.

She selects a group of 10 books to take to a book club meeting.

a. Find the probability that the group of 10 books consists of 2 adult fiction books, 3 adult non-fiction books and 5 children's books.

b. Find the probability that the group of 10 books selected contains 5 or 6 children's books.

**SECTION 8.4 CONDITIONAL PROBABILITY**  
**CONDITIONAL PROBABILITY RULE and MULTIPLICATION RULE**

**IF : CONDITIONAL PROBABILITY**

**Probability that event A occurs IF (given that) we know that outcome B has occurred**

$P(A|B)$  = Probability that event A occurs if we know that outcome B has occurred

$P(A|B)$  = Probability that event A occurs “given that” outcome B has occurred

The vertical line means “if” ; we can also say “given that”

- ◆ The event we are interested in comes appears before (to the left of) the “if line”
- ◆ The condition is the outcome we know about; it appears after (to the right of) the “if line”.

**The condition reduces the sample space to be smaller by eliminating outcomes that did not occur.**

**EXAMPLE 14:** Two coins are tossed.

Each coin is a fair coin and has equal probability of landing on Head (H) or Tail (T).  
 Sample space  $S = \{ HH, HT, TH, TT \}$

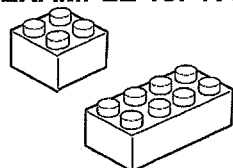


Find the probability of getting “two heads”.



Find the probability of getting “two heads” given that “at least one head” is obtained.

**EXAMPLE 15:** A box of 25 Lego blocks contains:



2 yellow square blocks  
 4 blue square blocks  
 4 green square blocks

3 yellow rectangular blocks  
 8 blue rectangular blocks  
 4 green rectangular blocks

Y: yellow
B: blue
G: green
S: square
R: rectangle

A child randomly selects one block at random.

Find  $P(B)$

Find  $P(B|S)$

Find  $P(S|B)$

Find  $P(Y)$

Find  $P(Y|S)$

**EXAMPLE 16:** A large car dealership examined a sample of vehicles sold or leased in the past year. Data is classified by type (**car, SUV, van, truck**) and by whether they were a sale of a **new** or **used** vehicle or whether the vehicle was **leased**.

	Car (C)	SUV (S)	Van (V)	Truck(T)	Total
New vehicle sale (N)	86	25	21	38	170
Used vehicle sale (U)	39	13	4	22	78
Vehicle Lease (L)	34	12	6	0	52
Total	159	50	31	60	300

Suppose a vehicle in the sample is randomly selected to review its sales or lease papers.

Find the probability that a vehicle was a Car IF (GIVEN THAT) it was Leased.

Find the probability that the vehicle was Leased IF (GIVEN THAT) it was a Car.

A contingency table displays data for two variables. This table shows the number of individuals or items in each category. We can use the data in the table to find probabilities.

**All probabilities EXCEPT conditional probabilities have the grand total in the denominator**

**Conditional Probabilities:** The condition limits you to a particular row or column in the table.

Condition says "IF" we look only at a particular row or column, find the probability

The **denominator will be the total for the row or column** in the table that corresponds to the condition

#### PROBABILITY RULES

◆ **Complement Rule:**  $P(E') = 1 - P(E)$

◆ **Addition Rule for OR (Union  $\cup$ ) Events:**  $P(E \cup F) = P(E) + P(F) - P(E \cap F)$   
 $P(E \text{ OR } F) = P(E) + P(F) - P(E \text{ AND } F)$

IF AND ONLY IF events are MUTUALLY EXCLUSIVE:  $P(A \text{ or } B) = P(A) + P(B)$

◆ **Multiplication Rule for AND (Intersection  $\cap$ ) Events:**  $P(E \text{ and } F) = P(E | F) P(F)$   
 $P(E \cap F) = P(E | F) P(F)$

IF AND ONLY IF events are INDEPENDENT (Sect. 8.5)  $P(E \text{ and } F) = P(E \cap F) = P(E) P(F)$

◆ **Conditional Probability Rule:**  $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \text{ AND } B)}{P(B)}$   
 ("given that", "if")

#### EXAMPLE 17: CONDITIONAL PROBABILITY RULE

In a certain town: 70% of households have Cable TV (*event C*)

55% of households have Netflix (*event N*)

These figures include the fact that 42% of households subscribe to both.

a. Find the probability that a household subscribes to Netflix **given that** the household has Cable TV

b. Find the probability that a household has Cable TV **if** the household subscribes to Netflix



**EXAMPLE 18: Multiplication Rule for  $\cap$  AND Events**

At a college: 45% of all students take Statistics.

70% of all students intend to Transfer.

Of the students who intend to Transfer, 60% of them take Statistics.

Find the probability that a student intends to Transfer AND takes Statistics.

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**EXAMPLE 19: Multiplication Rule for  $\cap$  AND Events**

At a fast food restaurant: 75% of customers order burgers (*event B*)

70% of customers order fries (*event F*)

Of the customers who order burgers, 80% also order fries.

Find the probability that a customer orders both a burger and fries.

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**EXAMPLE 20: OPTIONAL EXTRA PRACTICE: Multiplication Rule for  $\cap$  AND Events**

(Hint: Read carefully to understand information given in the "story")

For job listings on a job posting website : 30% require professional certification (*event C*)

65% require a college degree (*event D*)

50% require 5+ years of related job experience (*event E*)

14% of job listings requiring a college degree also require professional certification.

Find the probability that a job requires both professional certification and a college degree.

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**EXAMPLE 21: OPTIONAL EXTRA PRACTICE: CONDITIONAL PROBABILITY RULE**

Big Shoe Wearhouse is concerned about customer satisfaction with online purchases.

40% of all shoe sales are online on their website. 60% of all shoe sales are in their stores. Overall, 15% of all shoe purchases are returned. 10% of all shoe purchases were made on the website **and** were returned.

Events:  $S$  = purchased in store       $W$  = purchased on website       $R$  = item is returned

a. Find the probability that a shoe purchase was made on the website **given that** it is returned.

b. Find the probability that a shoe purchase is returned **if** it was purchased on the website

## SECTION 8.5: INDEPENDENT EVENTS

### INDEPENDENT EVENTS:

Two events are independent if and only if the probability of one event (E) occurring is not affected by whether the other event (F) occurs or not.

**Events E and F are independent if  $P(E|F) = P(E)$   
or equivalently, if  $P(F|E) = P(F)$**

- knowing that one occurs does not change the probability of the other occurring
- $P(\text{event} | \text{condition}) = P(\text{event})$

**EXAMPLE 22:** Source: <http://www.indexmundi.com/blog/index.php/2013/06/25/male-and-female-literacy-rates-by-country/>

In Argentina, the literacy rate is 97% for men and 97% for women.

The overall literacy rate is 97%.

**Is the literacy rate in Argentina independent of gender?** Justify your answer using appropriate probabilities.

Events:  $F = \text{female}$        $M = \text{male}$        $L = \text{literate}$

**EXAMPLE 23:** Source: <http://www.censusindia.gov.in/2011-prov-results/indiaatglance.html>

In India, literacy rates are 82.1% for men and 65.5% for women

The overall literacy rate is estimated as approximately 74%.

**Is the literacy rate in India independent of gender?** Justify your answer using appropriate probabilities.

Events:  $F = \text{female}$        $M = \text{male}$        $L = \text{literate}$

Note: The literacy rates in India have improved, overall, and particularly for females, the gap is closing:

2011 literacy rates: Overall 74%    Male: 82.1%    Female: 65.5%

2001 literacy rates: Overall 64.8%    Male: 75.3%    Female: 53.7%

### TO CHECK IF TWO EVENTS ARE INDEPENDENT in a word problem: TWO METHODS

- |   |   |
|---|---|
| <ul style="list-style-type: none"><li>• Identify the probabilities you are given by reading the problem carefully</li><li>• See which is a conditional probability:<br/><math>P(\text{event} \text{condition})</math></li><li>• Compare it to probability of same event without the condition: <math>P(\text{event})</math></li><li>• If <math>P(\text{event}) = P(\text{event} \text{condition})</math> events are independent</li></ul> | <ul style="list-style-type: none"><li>• Find <math>P(E \cap F)</math> from the information given in the problem</li><li>• Calculate <math>P(E)P(F)</math></li><li>• If <math>P(E \cap F) = P(E)P(F)</math> events are independent</li></ul> |
|---|---|

## SECTION 8.5: INDEPENDENCE EXAMPLES

### EXAMPLE 24:

The table below shows the distribution of color blind people by gender in a sample of 100 people.

	Male (M)	Female (F)	Total
Color Blind (C)	6	1	7
Not Color Blind (N)	46	47	93
	52	48	100

Are the events color blind and male independent?

**EXAMPLE 25:** In a city with 2 airports, 100 flights were surveyed and 20 of those departed late.  
Of the 45 flights in the survey from airport A, 9 departed late  
Of the 55 flights in the survey from airport B, 11 departed late  
Are the events “depart from airport A” and “departed late” independent?

**EXAMPLE 26:** The probability that Jaime will travel to visit his aunt is 0.30.  
The probability that he will go river rafting is 0.50.  
If the two events are independent, what is the probability that Jaime will do both?

### OPTIONAL EXTRA PRACTICE PROBLEMS FOR INDEPENDENCE IN CONTINGENCY TABLES

An easy way to check if two events are independent in a contingency table is

Let the column be the "condition"

Let the row be the "event"

**Compare :**  $P(\text{row event} \mid \text{condition in column})$  to  $P(\text{row event using total column})$

**If and only if these probabilities are equal, then the events are independent**

**EXAMPLE 27: OPTIONAL PRACTICE: Are the events N and V independent?**

	Car (C)	SUV (S)	Van (V)	Truck(T)	Total
New vehicle sale(N)	86	25	21	38	170
Used vehicle sale (U)	39	13	4	22	78
Vehicle Lease (L)	34	12	6	0	52
Total	159	50	31	60	300

Show your work to justify your answer using appropriate numerical evidence in the probabilities.

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**EXAMPLE 28: OPTIONAL PRACTICE: Are the events S and U independent?**

	Car (C)	SUV (S)	Van (V)	Truck(T)	Total
New vehicle sale(N)	86	25	21	38	170
Used vehicle sale (U)	39	13	4	22	78
Vehicle Lease (L)	34	12	6	0	52
Total	159	50	31	60	300

Show your work to justify your answer using appropriate numerical evidence in the probabilities.

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**EXAMPLE 29 OPTIONAL PRACTICE:**

Suppose that a sample residents of a town with a large university gave the data below:

	(S) College Student	(N) Not College Student	TOTAL
(A) Amazon Prime Member	40	20	60
(B) Not Amazon Prime Member	60	130	190
TOTAL	100	150	250

**Are events of "student" and "Amazon Prime member" independent?**

Show your work to justify your answer using appropriate numerical evidence in the probabilities.

## SECTION 9.2 BAYES THEOREM (flipping the tree)

**EXAMPLE 30** A certain virus infects 10% of people  
A test used to detect the virus can give a positive result or a negative result.  
The test results are positive 80% of the time IF the person has the virus  
For people who do not have the virus, the test results are positive 5% of the time (“false positive”)

**V** = event that a person has the virus

**Pos** = event that the test is positive      **Neg** = event that the test is negative

- a. Find the probability that a person tests positive and has the virus
- b. Find the probability that a person tests positive and does not have the virus
- c. Find the probability that a person tests positive
- d. Are the events “positive” and “has the virus” independent? Justify using probabilities.
- e. Find the probability that a person has the virus if they test positive

## SECTION 9.2 BAYES THEOREM (flipping the tree)

**EXAMPLE 31:** (this example is in section 9.2 of the textbook)

A department store buys 50% of its appliances from Manufacturer A, 30% from Manufacturer B, and 20% from Manufacturer C.

It is estimated that 6% of Manufacturer A's appliances, 5% of Manufacturer B's appliances, and 4% of Manufacturer C's appliances need repair before the warranty expires.

An appliance is chosen at random. If the appliance chosen needed repair before the warranty expired, what is the probability that the appliance was manufactured by Manufacturer A? Manufacturer B? Manufacturer C?

## SECTION 9.3 EXPECTED VALUE

If an experiment has numerical outcomes and it has a probability distribution,

Outcome Value of X	$x_1$	$x_2$	$x_3$	$\dots$	$x_n$
Probability	$p(x_1)$	$p(x_2)$	$p(x_3)$	$\dots$	$p(x_n)$

then the expected value of the experiment is

$$E = \text{Expected Value} = x_1p(x_1) + x_2p(x_2) + x_3p(x_3) + \dots + x_np(x_n)$$

**The expected value is interpreted as a long term or long run average.**

If you have taken Math 10 Statistics, you may have seen this as  $\mu = \sum x P(x)$   
 where the symbol  $\mu$  (Greek letter "mu") stands for "mean" or "expected value"

### EXAMPLE 32:

Students who live in the dormitories at a certain four year college must buy a meal plan. They must select from four available meal plans: 10 meals, 14 meals, 18 meals, or 21 meals per week. The Food and Housing Office has determined that 15% of students purchase 10 meal plan, 45% purchase the 14 meal plan of students, 30% purchase the 18 meal plan ,10% purchase the 21 meal plan.  
 $X$  = the number of weekly meals that an individual student purchase on their meal plan

Notation:  $P(\text{Event}) = \text{probability value}$

- $P(X = 10)$  is the probability that a student purchases a meal plan with 10 meals per week
- $P(X > 14)$  is the probability that a student purchases a meal plan with more than 14 meals per week
- $P(X \leq 18)$  is the probability that a student purchases a meal plan at most 18 meals per week

$x$ =Number of Meals	Probability $P(x)$
<b>10</b>	
<b>14</b>	
<b>18</b>	
<b>21</b>	

This table is called the PDF  
 Probability Distribution Function

On average, how many meals does a student purchase per week in their meal plan?

If there are 2000 students living in the dorm, how many meals do does the dining hall expect to serve each week?

### EXAMPLE 33: Keisha makes and sells jewelry at crafts fairs during on summer weekends

If the weather is hot and sunny, she earns a profit of \$1000. If the weather is cool and sunny, she earns a profit of \$2000. If the weather is rainy she loses \$400 because she pays booth rental and travel costs but does not sell much jewelry in the rain. There is 30% chance the weather is hot and sunny, a 50% chance it is cool and sunny and a 20% chance that it rains. Find and interpret the expected value.

## SECTION 9.3 EXPECTED VALUE

### EXAMPLE 34 :

A real estate developer is presenting plans to the Planning Commissioner for a development of houses and apartments he proposes to build. He needs to estimate the impact on the local schools so he must estimate the number of children expected to attend the schools. He hires a statistician who studies the demographics of the neighborhood and of similar housing developments; she provides the estimates in the table.

Let  $X$  = the number of school age children per household.

$X$	$P(X)$
0	0.30
1	0.20
2	
3	0.18
4	0.04
5	0.01
6 or more	0

- Find the probability that a household has 2 school age children
- Find the probability that a family has **at most 3** school age children.
- Find and interpret the **expected number** of school age children per household.
- Find the expected total number of school age children in this development if 120 housing units are built.

**Extra Practice Examples:** The following two examples are from the textbook. Try these questions yourself for practice and Read Section 9.3 in the textbook to see solutions to these two examples.

### EXAMPLE 35 : Section 9.2 Example 2 from Textbook

To sell an average house, a real estate broker spends \$1200 for advertisement expenses. If the house sells in three months, the broker makes \$8,000. Otherwise, the broker loses the listing. If there is a 40% chance that the house will sell in three months, what is the expected payoff for the real estate broker?

### EXAMPLE 36 : Section 9.2 Example 4 from Textbook

A lottery consists of choosing 6 numbers from a total of 51 numbers. The person who matches all six numbers wins \$2 million. If the lottery ticket costs \$1, what is the expected payoff?



## SECTION 9.1 : BINOMIAL PROBABILITY DISTRIBUTION

A BINOMIAL probability experiment has

- a fixed number  $n$  of repeated trials
- each trial has outcomes that we can classify as “success or “failure”
- outcome of trials are independent (*Outcome of a trial does not influence outcome of future trials*)
- the probability of success on a single trial,  $p$ , is constant (the same) for all trials

**We are interested in the number of successes,  $x$ , in  $n$  trials**

**EXAMPLE 37:** A college claims that 70% of students receive financial aid. Suppose that 4 students at the college are randomly selected. We are interested in the number of students in the sample who receive financial aid.

$X =$  \_\_\_\_\_  
 $p =$  the probability that a student receives financial aid:  $p =$  \_\_\_\_\_  $q = 1-p =$  \_\_\_\_\_

X	P(x)
0	
1	
2	
3	
4	

Ways to get  $x$  successes in  $n$  trials

$n = 4$ $x = 1$ Abcd aBcd abCd abcd	$n = 4$ $x = 2$ ABcd AbCd AbcD aBCd aBcD abCD	$n = 4$ $x = 3$ aBCD AbCD ABcD ABCD
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Find the probability that AT MOST 2 of the students in the sample receive financial aid:

**EXAMPLE 38:** An auto insurance company estimates that 15% of its auto insurance claims involve fraud. An auditor randomly selects 12 claims out of a huge population of tens of thousands of claims to review. Show work using the binomial probability formulas.

- a. Find the probability that 2 of the 12 selected claims involve fraud.
- b. Find the probability that half of the 12 selected claims involve fraud.
- c. Find the probability that 8 of the 12 selected claims involve fraud.
- c. Find the probability that none of the 12 selected claims involve fraud.
- d. Find the probability that at least one claim involves fraud.

**Formulas for Binomial Distribution:**  $P(X = x) = {}_n C_x p^x (1-p)^{n-x}$

$P(X = x)$  is the probability of obtaining  $x$  successes in  $n$  independent trials

$$P(X = x) = \left( \begin{matrix} \text{Number of ways to get} \\ x \text{ successes in } n \text{ trials} \end{matrix} \right) \left( \begin{matrix} \text{probability} \\ \text{of success} \end{matrix} \right)^{\text{number of successes}} \left( \begin{matrix} \text{probability} \\ \text{of failure} \end{matrix} \right)^{\text{number of failures}}$$

**ADDITIONAL PRACTICE PROBLEM FOR BINOMIAL PROBABILITY**

**EXAMPLE 39 :**

[http://www.pewresearch.org/fact-tank/2016/01/05/pew-research-center-will-call-75-cellphones-for-surveys-in-2016/?utm\\_source=Pew+Research+Center&utm\\_campaign=4a62041804-Methods](http://www.pewresearch.org/fact-tank/2016/01/05/pew-research-center-will-call-75-cellphones-for-surveys-in-2016/?utm_source=Pew+Research+Center&utm_campaign=4a62041804-Methods) Newsletter for June 6 24 2015

A Pew Research Center study of phone ownership cites that:

65.7% of 25- to 29-year-olds live in wireless-only households, that is, own a cell phone only and do not have landline phones.

a. Find the probability that in a sample of 10 people age 25-29, that 4 of the people in the sample have landlines.

b. Find the probability that in a sample of 10 people age 25-29, that only 1 person does not have a landline.