Electric Potential Energy

Recall that in PHYS 4A using the concept of potential energy and conservation of energy provided a much more convenient (simpler) method of describing the motion of systems. This method was easier than using forces and N2L. Likewise, in our study of electricity, also using the concept of electric potential energy and conservation of energy will also provide a much more convenient (simpler) method of describing the motion of electric charges. This method will also be easier than using Coulomb's Law and N2L. But first let's compare the force of gravity to the electric force and note the similarities:

1. Law of Gravity

$$\boxed{\vec{F}_g = \frac{Gm_1m_2}{r^2}\hat{r}}$$

2. Gravitational Field

$$\boxed{\vec{g} = \frac{\vec{F}_g}{m_o}}$$

3. Gravitational Force is a conservative force.

$$\boxed{w_g = \int_a^b \vec{F}_g \bullet d\vec{\ell}}$$ Path Independent

$$\boxed{w_g = \oint \vec{F}_g \bullet d\vec{\ell} = 0}$$

$$\boxed{w_g = -\Delta U_g}$$

4. Gravitational Potential Energy

$$\boxed{U_g = -\frac{Gm_1m_2}{r}}$$

1. Coulomb's Law

$$\boxed{\vec{F}_E = \frac{kq_1q_2}{r^2}\hat{r}}$$

2. Electric Field

$$\boxed{\vec{E} = \frac{\vec{F}_E}{q_o}}$$

3. Electric Force is a conservative force

$$\boxed{w_E = \int_a^b \vec{F}_E \bullet d\vec{\ell}}$$ Path Independent

$$\boxed{w_E = \oint \vec{F}_E \bullet d\vec{\ell} = 0}$$

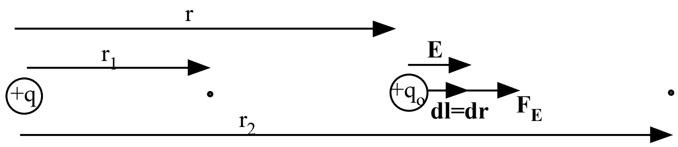
$$\boxed{w_E = -\Delta U_E}$$

4. Electric Potential Energy

$$\boxed{U_E = \frac{kq_1q_2}{r}}$$

Electric Potential Energy

To obtain the equation for the electric potential energy function between two point charges let's find the work done in moving a charge q_o between two points in the E-field produced by a point charge *q*. We will assume for now that both charges are positive but our result will be valid in general.



$$w_E = \int_{r_1}^{r_2} \vec{F}_E \bullet d\vec{r} = \int_{r_1}^{r_2} \frac{kqq_o}{r^2}\, dr = kqq_o \left(-\frac{1}{r} \right) \Bigg|_{r_1}^{r_2}$$

$$\boxed{w_E = \frac{kqq_o}{r_1} - \frac{kqq_o}{r_2}}$$ Work Done by Electric Force

Since the electric force is conservative, then:

$$w_E = -\Delta U_E$$
$$\Delta U_E = -w_E$$
$$U_2 - U_1 = \frac{kqq_o}{r_2} - \frac{kqq_o}{r_1}$$

As always the choice of the zero point of potential energy is completely arbitrary. For convenience we choose U = 0 when the force between the point charges is zero. Thus $U_1 = 0$ when $r_1 \to \infty$.

$$U_2 - 0 = \lim_{r_1 \to \infty} \left(\frac{kqq_o}{r_2} - \frac{kqq_o}{r_1} \right)$$

$$\boxed{U = \frac{kqq_o}{r}}$$ Electric Potential Energy

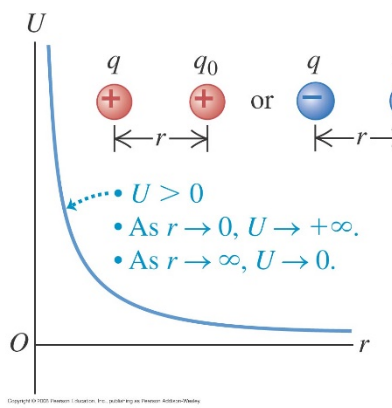
Notice that this equation is similar to $U = -\frac{Gm_1 m_2}{r^2}$ for gravitational PE where the negative is due to the fact that the gravitational force is attractive.

This equation is valid regardless of the sign of the charges.

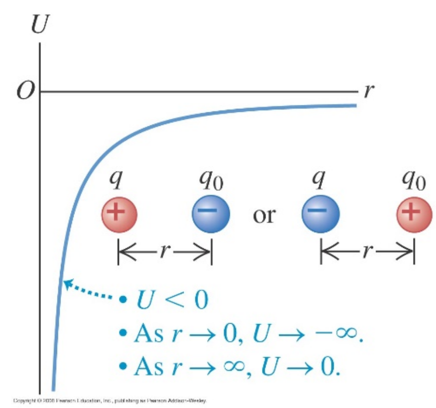
1. U > 0 if q and q_o have the same sign.
2. U < 0 if q and q_o have opposite sign.

$$\boxed{U = \frac{kqq_o}{r}}$$ Electric Potential Energy

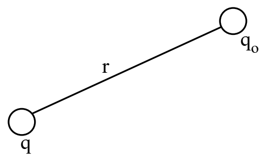
(a) q and q_0 have the same sign.



(b) q and q_0 have opposite signs.

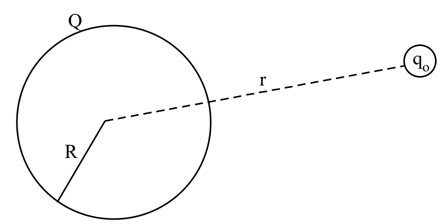


Ex.



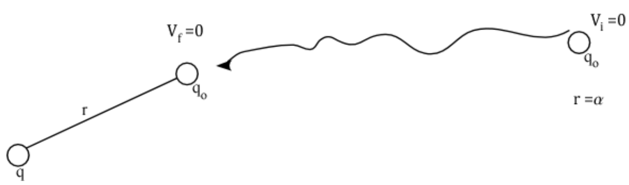
$$U = \frac{kqq_o}{r}$$

Ex. Sphere with uniform volume charge density ρ



$$U = \frac{kQq_o}{r} \quad (r \geq R)$$

Now consider a point charge q at rest and a second point charge q_o at rest at infinity. What is the work required to bring charge q_o from infinity to a distance *r* from q?

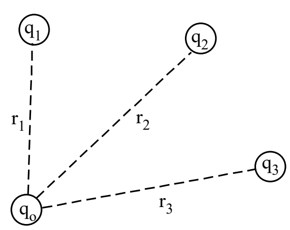


Recall that the work done on a system by an external agent is given by
$W_{ext} = \Delta U + \Delta K + \Delta E_{\text{int}}$

$$W_{ext} = U_f - U_i = \frac{kqq_o}{r} = U$$
$$W_{ext} = U = \frac{kqq_o}{r}$$

U = work done by an external agent to bring q_o from infinity to its final position *r*.

Find the potential energy of the charge q_o in the following charge configuration.



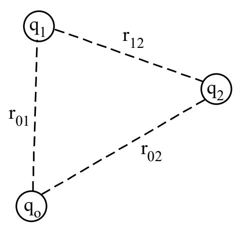
$$U = \frac{kq_oq_1}{r_1} + \frac{kq_oq_2}{r_2} + \frac{kq_oq_3}{r_3}$$

This potential energy is equal to the work done by an external agent in bringing the charge q_o from infinity to its final position. The work is done against the electric force (electric field) due to all the charges.

In general,

$$\boxed{U = kq_o \sum_i \frac{q_i}{r_1}}$$ Potential Energy of a Point Charge q_o due to interaction with surrounding charges.

Find the total potential energy of the following system.



Consider assembling the charges in the system when the charges are initially separated by an infinite distance apart.

$$\boxed{U_{system} = \frac{kq_o q_1}{r_{01}} + \left(\frac{kq_o q_2}{r_{02}} + \frac{kq_1 q_2}{r_{12}} \right)}$$ Potential Energy of a 3-point charge system

The potential energy of the system is equal to the work done by an external agent in bringing the charges from infinite separation to their final positions. The work is done against the electric force due to the present charges.

In calculating the total electric potential energy of a system you need to take into account the interaction due to ALL pairs of charges. In calculating the change in PE of a system you need to only take into account the pairs of charges whose distances are changing. If the distance between pairs of charges do not change, then their PE doesn't change and will cancel out before and after!